

Wave Mechanics of Risk

A New Paradigm for Actuarial Science

Foundations, Theory, and Applications to
Catastrophe Risk Pricing and Insurance

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First Edition — 2025

SEVEN LIONS EDITIONS

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First Edition

ISBN 979-8-19-538645-0

A companion website for this book is available at quantum-actuarial-science.com, providing additional resources, worked examples, and updates.

The opinions expressed in this work are those of the author and do not necessarily represent the views of any institution or professional body.

*To Louis de Broglie, whose audacity showed that
the most abstract mathematical wave
can illuminate the deepest physical reality.*

*And to the actuaries of tomorrow,
who will dare to see risk as it truly is:
not a number, but a wave.*

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Wave Mechanics of Risk

A NEW PARADIGM FOR ACTUARIAL SCIENCE

The standard tools of actuarial science — Gaussian distributions, linear correlations, static exposure models — were designed for a world that does not exist. Insurance exists precisely to confront a far more complex reality: contagion, structural breaks, feedback effects, and rare events that redefine entire systems.

This book proposes a fundamental paradigm shift. Drawing on the wave-particle duality of Louis de Broglie and the wave equation of Erwin Schrödinger, it develops a complete theoretical framework in which every risk carries a wave function. Risks interfere and superpose; catastrophes propagate as shock waves; the uncertainty principle governs the fundamental limits of actuarial precision; and quantum tunnelling models the reinsurance of tail events that classical theory deems impossible.

Grounded in physics and calibrated to real insurance data, *Wave Mechanics of Risk* offers practitioners and researchers the mathematical foundations of a new science of catastrophe.

Topics covered

De Broglie–Schrödinger formalism • Wave function of risk • Risk interference & superposition • Actuarial uncertainty principle • Energy-level pricing • Wave-based IBNR reserves • Reinsurance & quantum tunnelling • Portfolio decoherence • Catastrophe shock waves • Cat Nat pricing • Climate change drift

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